This .pdf file puts together two corrections on the following article:

Georgios Gerasimou (2021) “Simple preference intensity comparisons”, *Journal of Economic Theory*, 192, 105199. doi.org/10.1016/j.jet.2021.105199

Correction 1:
Georgios Gerasimou (2022) “Corrigendum to Gerasimou (2021) and comment on Banerjee (2022)”, *Journal of Economic Theory*, 205, 105542. doi.org/10.1016/j.jet.2022.105542

Correction 2:
This is unpublished, dated July 2024, and pertains to Example 2 & Figure 1 on pp. 8-9.

Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 205 (2022) 105542
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Corrigendum

Corrigendum to Gerasimou (2021) and comment on Banerjee (2022)

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Received 17 August 2022; accepted 19 August 2022

Abstract

We clarify that Theorem 1 in Gerasimou (2021) is correct when the following minor changes are made to the definitions of two of its components: (1) the Translocation Consistency axiom requires an equivalence rather than a one-way implication; (2) a strict inequality on the left hand side of the "lateral consistency" property of preference intensity functions implies a strict inequality on its right hand side. We also clarify that the additional preference intensity orderings allowed for by Banerjee's (2022) generalization of Theorem 1, which replaces Translocation Consistency with an axiomatized version of lateral consistency, imply the existence of at least three alternatives a, b, c where: (i) a is preferred to b is preferred to c; (ii) one or both of these preferences is strict; (iii) a is strictly preferred to c exactly as much as a is to b or b is to c.

JEL classification: D01; D11; D60; D90

We start with the complete statement of the result in question:

Theorem 1 in Gerasimou (2021). *The following are equivalent for a binary relation $\dot{\succsim}$ on a finite set $X \times X$:*

1. *$\dot{\succsim}$ satisfies Weak Order, Reversal and Translocation Consistency.*
2. *$\dot{\succsim}$ satisfies Weak Order, Reversal and Separability.*
3. *$\dot{\succsim}$ is representable by an odd-ordinally unique preference intensity function.*
4. *$\dot{\succsim}$ is representable by an ordinally unique general preference intensity function.*

DOI of original article: https://doi.org/10.1016/j.jet.2021.105199.
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[1] I gratefully acknowledge helpful remarks by Matúš Tejiščák on Claim 2 in Gerasimou (2021). I also declare that I have been in no direct correspondence with the author of Banerjee (2022) to date. First version: 24 February 2022.

https://doi.org/10.1016/j.jet.2022.105542
0022-0531/

We will first show that this result is correct as stated once the following two minor changes are made to the definitions of two of its components:

(1) **Translocation Consistency** requires an equivalence rather than a one-way implication: *For all* $a, b, c \in X$, $(a, c) \dot{\succsim} (b, c) \Longleftrightarrow (a, b) \dot{\succsim} (b, a)$.
(2) When the left-hand-side inequality in (5c) is strict, so is its right-hand-side inequality. [Analogously for (4c)].

To do so, we begin by completing the proof of Claim 2 using (1). The statement and original proof of this claim are reproduced verbatim below, except for the inserted additional text in square brackets which makes use of the converse implication in (1).

Claim 2. *If* $\dot{\succsim}$ *satisfies Weak Order, Reversal and Translocation Consistency, then* $\succsim$ *is a weak order on* X.

Completeness of $\succsim$ immediately follows from its definition and the assumed Completeness of $\dot{\succsim}$. For Transitivity, suppose $a \succsim b \succsim c$ and assume to the contrary that $c \succ a$. We have $(a, b) \dot{\succsim} (b, a)$, $(b, c) \dot{\succsim} (c, b)$ and $(c, a) \dot{\succ} (a, c)$. In view of Claim 1, $(a, b) \dot{\succsim} (b, a)$ and $(c, a) \dot{\succ} (a, c)$ implies $(a, b) \dot{\succ} (a, c)$. By Reversal, $(c, a) \dot{\succ} (b, a)$. By Translocation Consistency, $\Big[$ $(c, b) \dot{\succsim} (b, c)$. Suppose to the contrary that $(c, b) \dot{\sim} (b, c)$. By Translocation Consistency, this implies $(c, a) \dot{\sim} (b, a)$. By Reversal, this is equivalent to $(a, b) \dot{\sim} (a, c)$, which contradicts the conclusion $(a, b) \dot{\succ} (a, c)$ above. It follows that $\Big]$ $(c, b) \dot{\succ} (b, c)$. This contradicts the postulate $(b, c) \dot{\succsim} (c, b)$. $\square$

Next, although various parts of the "4 ⇒ 1" step in the proof of Theorem 1 in Gerasimou (2021) already assume (2), the paragraph below must be added at the end of that proof to show that both implications of Translocation Consistency follow from (5a)-(5c):

To establish the converse implication of Translocation Consistency, suppose $(a, b) \dot{\succsim} (b, a)$ holds and assume to the contrary that there is $c \in X$ such that $(b, c) \dot{\succ} (a, c)$. We have $g(b, c) > g(a, c)$ and $g(a, b) \geq g(a, a) \geq g(b, a)$. Consider first the case where $g(b, c) \geq g(c, c)$. The above imply $\min\{g(a, b), g(b, c)\} \geq g(a, a) = g(c, c)$ and $g(a, c) < g(b, c)$, which contradicts (5c). Now consider $g(c, c) > g(b, c)$. We have $g(a, b) \geq g(a, a) = g(c, c) > g(b, c) > g(a, c)$. By (5b), $g(c, a) > g(c, b) > g(c, c) = g(a, a)$. Thus, $\min\{g(c, a), g(a, b)\} \geq g(a, a) = g(c, c)$ and $g(c, b) < g(c, a)$, which also contradicts (5c). Therefore, $(a, b) \dot{\succsim} (b, a)$ implies $(a, c) \dot{\succsim} (b, c)$ for all $c \in X$. $\square$

We now turn to Banerjee's (2022) comment. His example function below shows that neither Translocation Consistency nor Separability are necessary for a preference intensity relation that satisfies Weak Order and Reversal to be representable by a preference intensity function that may violate (2):

$$U(a, c) = U(b, c) = 7, \; U(b, a) = 6, \; U(x, y) = -U(y, x) \text{ for } x, y \in \{a, b, c\}.$$

Indeed, this function has the property

$$U(b, a) = \min\{U(b, a), U(a, c)\} = 6 \quad \text{and}$$
$$U(b, c) = U(a, c) = \max\{U(b, a), U(a, c)\} = 7.$$

As was clarified above, this is consistent with Theorem 1 in Gerasimou (2021) when (1) and (2) are assumed explicitly, as per the original paper's intention.

In his Theorem 1, moreover, and deploying an argument that follows closely the one in the proof of Theorem 1 in Gerasimou (2021), Banerjee (2022) extends the latter result with a representation that does not require (2) and can be characterized by replacing Translocation Consistency (or Separability) with an axiomatized version of the "lateral consistency" property that was introduced in Gerasimou (2021) for preference intensity functions.

It is immediate that the classes of relations $\dot{\succsim}$ that are encompassed by Theorem 1 in Gerasimou (2021) and Theorem 1 in Banerjee (2022) differ because the latter also includes those relations for which one of the following is necessarily true for at least one triple of alternatives $a, b, c \in X$:

$$(a,b) \dot{\sim} (c,c),\ \ (b,c) \dot{\succ} (c,c) \text{ and } (a,c) \dot{\sim} (b,c);$$
$$(a,b) \dot{\succ} (c,c),\ \ (b,c) \dot{\sim} (c,c) \text{ and } (a,c) \dot{\sim} (a,b);$$
$$(a,b) \dot{\succ} (c,c),\ \ (b,c) \dot{\succ} (c,c) \text{ and } (a,c) \dot{\sim} (a,b) \text{ if } (a,b) \dot{\succsim} (b,c);$$
$$(a,b) \dot{\succ} (c,c), (b,c) \dot{\succ} (c,c) \text{ and } (a,c) \dot{\sim} (b,c) \text{ if } (b,c) \dot{\succsim} (a,b).$$

Therefore, the additional intensity relations $\dot{\succsim}$ that are allowed for in Banerjee (2022) consist of those where there exist $a, b, c \in X$ such that:

(i) $a \succsim b \succsim c$;
(ii) one or both of these preferences is strict;
(iii) the decision maker strictly prefers a to c *exactly* as much as they prefer a to b or b to c.

It follows from the preceding analysis that these are precisely the intensity relations where violations of Translocation Consistency (or Separability) will occur.

References

Banerjee, K., 2022. Corrigendum on "Simple preference intensity comparisons". J. Econ. Theory 204, 105519.
Gerasimou, G., 2021. Simple preference intensity comparisons. J. Econ. Theory 192, 105199.

Simple Preference Intensity Comparisons: A Correction

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13 July 2024

The final sentence in the discussion of Example 2 (p.8) in Gerasimou (2021) contains a false claim: *"Finally, Fig. 1 shows that the preferences $\succsim_1$ and $\succsim_2$ induced by the intensities $\dot{\succsim}_1$ and $\dot{\succsim}_2$ that are represented by s_1 and s_2 are generally distinct, which in turn implies that $\dot{\succsim}_1$ and $\dot{\succsim}_2$ are themselves distinct."* More specifically, while it is correct that $\dot{\succsim}_1$ and $\dot{\succsim}_2$ are distinct, it is false that $\succsim_1$ and $\succsim_2$ are as well. The latter counter-claim is confirmed by observing that $s_2(a,b) \equiv s_2(a_1,a_2,b_1,b_2) := \frac{a_1}{b_1} - \frac{b_1}{a_1} + \frac{a_2}{b_2} - \frac{b_2}{a_2} = 0 \Leftrightarrow a_1a_2 = b_1b_2 \Leftrightarrow a \sim_2 b$. Thus, the preferences induced by s_2 are symmetric Cobb-Douglas and coincide with those induced by $s_1(a_1,a_2,b_1,b_2) := a_1a_2 - b_1b_2$. (The solid curve in Fig. 1 erroneously depicts the set of all bundles $(x_1,x_2) \in \mathbb{R}^2_{++}$ such that $s_2(x_1,x_2,10,10) = 5$.) That the intensity relations induced by $\dot{\succsim}_1$ and $\dot{\succsim}_2$ are distinct as claimed, however, is verifiable by contrasting their values e.g. at $a := (20,19)$, $b := (18,17)$, $c := (21,20)$, $d := (19,18)$: $s_1(a,b) = 74 < 78 = s_1(c,d)$ and $s_2(a,b) \approx 0.43 > 0.41 \approx s_2(c,d)$. Thus, $(c,d) \mathrel{\dot{\succ}_1} (a,b)$ and $(a,b) \mathrel{\dot{\succ}_2} (c,d)$. A corrected last sentence on p.8 would therefore read as follows: *"Finally, while the preferences $\succsim_1$ and $\succsim_2$ induced by the intensities $\dot{\succsim}_1$ and $\dot{\succsim}_2$ that are represented by s_1 and s_2 coincide, the intensities $\dot{\succsim}_1$ and $\dot{\succsim}_2$ are generally distinct."*

References

Gerasimou, G. (2021): "Simple Preference Intensity Comparisons," *Journal of Economic Theory*, 192, 105199.